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Knowledge Construction Schemata of Teachers in Solving Real World Non-Routine Problem Situation: Their Implications to Mathematics Education

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Abstract

The study investigated the nature of Knowledge Construction Schemata (KCS) that teacher-solvers use to solve a real-world non-routine problem situation. Eighteen Math teachers in different schools of Region I and the Cordillera Administrative Region were given a carefully selected power problem, which they solved in at most two hours. Results showed that rigid procedural framework of thought characterizes respondents' KCS in solving problems. Based on this framework, solvers see solutions to a problem situation as purely routine or algorithmic procedures, a condition that makes them selective in interpreting data. They give meaning only to quantitative data while ignoring the qualitative ones, resulting in incomplete solution steps and failure to solve the problem. The influence of the routine type of problem solving appears to be so entrenched that solvers could not find meaning in qualitative data and venture to alternative solution steps that do not necessarily address the problem situation. An important component of problem solving, which is making necessary adjustments in response to a new problem situation (accommodation process), remains a great challenge among the teacher-solvers. Their KCS nature is heavily confined to assimilation processes, which seem responsible for keeping solvers from making exploratory attempts that could have paved the way for more productive problem solving. The study recommends that real-world non-routine type of problem-solving be integrated with school mathematics to develop among the students flexible, reflective, and transformational KCS.

Introduction

One important issue in the Philippine educational system is whether it is imbuing in students a kind of knowledge construction schemata (or mathematical thinking patterns) that enable them to effectively deal with realworld problem situations, which could be solved mathematically. This question continues to steer the top management of the Philippine educational system to design a mathematics curriculum that could solve the country's challenge to train students to be at par in competence with those being trained by its neighboring countries. There are evidences that the Philippines is far behind

in Mathematics education. For example, it consistently placed third from the bottom among 48 countries in the 1999 and 2003 guadrennial mathematics and science comparative studies known as "Trend in International Mathematics and Science Study (TIMSS) (Orleans, 2007). Also, the math scores of the Philippine participants are 380 for fourth graders and 393 for eight graders, which are both way below international averages of 495 and 467, respectively (TIMMS & Pirls International Study Center, 2003). Since then, the Philippines never participated again, and so there is no way of knowing whether it has improved or remained in its previous standing visa-vis other countries. The impact of the TIMSS results appears to have sparked new motivations to revise the basic mathematics education curricula and pedagogies, which is why different ideas gleaned from around the world have found their way to the country's educational system. This predicament has led to the current efforts of shaping and reshaping the mathematics curriculum.

Amidst the above development, the researcher finds it necessary to determine the nature of the knowledge construction schemata of teachers in solving problems since they influence the formation of problem solving schemata of their students. Perhaps, part of the answer to the question of why Filipino students could hardly perform in mathematics and sciences at a high level and at par with students of its neighboring countries is not only due to the curriculum but also and, more importantly, due to the teachers themselves. The cliché is true that teachers cannot expect their students to develop good knowledge construction schemata in solving problems while they are maybe deficient in such. Thus, this study was conducted to input data from the teachers' perspective in understanding the issues surrounding students' performance in mathematics, particularly problem solving. The study attempted to seek answers within the school context to some of the persisting issues that beset mathematics education in the Philippines. It aimed to determine and characterize the nature of the mathematics teachers' KCS in interpreting and solving a realworld non-routine problem situation. Specifically, it tried to determine the level of the solvers' problem solving engagements in view of Polya's problem solving principles. It also determined the nature of the solvers' cognitive engagements in view of theory of cognitive development, Piaget's particularly adaptation processes.

Conceptual Framework

This study is guided by Jean Piaget's theory of cognitive development (Cherry, 2016; McLeod, 2018), constructivist theory of learning (Hein, 1991), and Polya's (1957) problem solving heuristics.

Two of the three important basic components of Piaget's theory that are considered in the study include schemas and adaptation processes (McLeod, 2018). First, schemas or schemata are defined as building blocks of knowledge with which people are able to interpret objects, actions, events, and abstract concepts. These are like "index cards filed in the brain, each one telling an individual how to react to incoming stimuli or information" (Wadsworth, 2004). These constitute people's mental representations of the world, which they use to explain the "whats and whys" of the things they perceive around them and provide a framework for understanding future events. Schemata are described as "patterns of thought or behavior that organize categories of information and the relationships among them" (DiMaggio, 1997).

Schemata play an essential role in terms of attention to stimuli and absorption of new knowledge. It is said that "people are more likely to notice things that fit into their schema while re-interpreting contradictions to the schema as exceptions or distorting them to fit" (Nadkarni & Narayanan, 2007).

While schemata are important in one's ability to make coherent perceptions of anything encountered in this world, once entrenched, these are hard to change. Belief systems such as religious beliefs, personal bias, stereotypes, and other preconceived ideas toward something are examples of schemata that are hard to change. Consequently, people holding certain schemata toward some particular issues continue to reason out consistent with said schemata even in the face of a reality that is incongruent with such frameworks of thought. In this case, the schemata that people hold for something may keep them from properly addressing situations or correctly solving problems.

The other component is the adaptation processes. "Adaptation is the ability to adjust to new information and experiences. Through

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adaptation, people are able to adopt new behaviors that allow them to cope with change" (Cherry, 2016). There are three components of adaptation processes that people experience when exposed to new information or faced with new situations. These include assimilation, accommodation, and equilibrium.

Assimilation is the process in which information coming from the outside world is transformed to fit a person's existing ideas and concepts, or schemata. When new information fit such schemata, these are readily assimilated. This is why people adopt behaviors in response to familiar situations more readily than unfamiliar situations (McLeod, 2018; Cherry, 2017).

Accommodation is the process in which existing schemata are modified, or even changed, to fit new information. Accommodation happens when existing schemata are insufficient to explain new phenomena or to deal with new situations. It is said that it is through the accommodation process that learning takes place, and consequently, cognitive development happens. Accommodation is a more difficult process because it entails adjustment, which may require changing existing schemata that may revolutionize long-held ideas and dislodge the person from his/her tightly held comfort zone (McLeod, 2018; Cherry, 2017).

The third component is the equilibrium. This aspect of adaptation happens if existing schemata are sufficient to explain new situations. Equilibrium brings a sense of stability to people's perception of things. In equilibrium, people adapt to new situations through assimilation by simply incorporating new information to their existing schemata about the things they encounter in life.

Another learning theory that guided the study is constructivism. According to this theory, "learners construct knowledge for themselves – each learner individually and socially constructs meaning- as he/ she learns. Constructing meaning is learning" (Hein, 1991). For knowledge construction to happen either through assimilation or accommodation or both processes, "active learners are required, not the passive ones, because problem solving skills cannot be taught but must be discovered" (Piaget & Inhelder, 1958).

One important learning activity in which learners demonstrate knowledge construction

is problem solving. This activity involves four basic components: "understanding the problem, planning, implementing the plan, and looking back," Polya (1957). Understanding the problem involves meaning making in which the solver makes sense of the problem situation. Planning involves framing workable solution steps based on existing knowledge structure, or schema, about how problems should be solved. It is in this stage that a solver's nature of adaptation process is manifested, that is, whether he/ she is simply assimilating or accommodating. Assimilating process is manifested if the solver is pursuing solution steps based purely on some established procedure of tackling a particular task. Accommodation process is manifested if a solver is framing original solution steps based on the specific context of the problem situation. Strategy implementation involves ensuring that solutions steps are knit together logically such that every step directly follows from previous steps, and computations are accurate. Looking back involves ascertaining that all data and conditions are considered in the solution.

Thus, to determine the nature of KCS of the respondents, a problem situation that requires active exploratory attempts and allows multiple solution paths must be given.

Methodology

The study is mainly qualitative. It used a descriptive approach in generating data consistent with the data analysis implied in Polya's problem solving heuristics (Polya, 1957) and Piaget's theory of cognitive development, particularly adaptation processes (Piaget, 1958). The data gathered were the solution steps or procedures implemented by the respondents in solving a real-world non-routine problem situation.

The respondents included 18 Mathematics graduate students enrolled during the second semester of SY 2016 – 2017. The respondents are a mix of full time and part-time teachers from different schools in the Cordillera Administrative Region (CAR) and Region I. Since this research is qualitative, the primary consideration in determining the number of respondents is the number who can saturate the data. In the study, 36

the first ten respondents already provided all possible solutions that could be done based on a particular framework of thought, and the rest just repeated what the first ten respondents provided. Given this, the researcher believed that the 18 respondents were more than enough to saturate the data. Moreover, since the respondents come from various schools (both public and private) of CAR and Region 1 and given that the learning goals and competencies for both basic Mathematics and tertiary Mathematics education as prescribed by the Department of Education (DepEd) and the Commission of Higher Education (CHED) are the same for all schools in the country, the data gathered from the respondents are believed to be reflective of the kind of thinking framework that the Philippine educational system has developed not only among the respondents of the study but also among its graduates across the country.

The outputs were grouped according to the nature of constructions manifested. Each group of outputs was analyzed in the light of the adaptation process involved to generate information about the kind of problem solving that respondents are capable of doing, the frameworks that direct their solution steps, and the struggles they are presently in. The respondents were given the following carefully selected problem real-world non-routine situation (Figure 1) which they solved in at most two hours.

The problem was tried out to a group of students (third year Math majors) not included in the study to determine whether this can elicit some exploratory attempts and varied KCS, which are characteristics primarily considered in choosing a problem. The basis for choosing these participants is their being a more mature group than the lower year level. Moreover, they were more readily available than the graduating group who were already sent out of the campus for their practice teaching. Also, the group has already taken almost all of their entire major subjects, thus making them able to tackle more difficult problems. Majority of the participants in the tryout group used ratio and proportion to find the distance that can be covered with 150 liters of gas based on the consumption rate of the vehicle. Other solutions provided by some participants were either not responsive to the problem or were illogically constructed. Despite the results, the problem was still considered because of its potential of being extended to various exploratory attempts, which are expected to happen if this is handled by math teachers.

The choice of the problem is consistent with what the mathematician and researcher Schoenfeld (1994), said that good mathematics problems can be extended for exploratory attempts by the solvers. The problem is considered a power problem and considering the amount of time expended to solve it, answers generated may

Figure 1

Real-world Non–routine Problem Situation Given to the Respondents

A farmer is to travel 150km using his old service jeep from his farm to his house in the city. The jeep is fully loaded with bananas and vegetables and it consumes 3 liters of gasoline for every 2 km travelled. The maximum amount of gas that the jeep's gas tank can accommodate above the minimum level that barely makes the engine start is 150 liters. The farmer has a total gas reserve of 450 liters stored in a small drum which cannot be brought along except if it is empty. The farmer can bring his two plastic basins which may be hanged at the rear end of the jeep. Each basin can contain 100 liters of gas for storage purposes only. What is the farthest distance that the farmer can travel? Will he be able to reach his house in the city?



Note: Photo adapted at World of Preposterously Overloaded Vehicles. *Chill Out Point*. Copyright 2020 by Chill Out Point.

be regarded as close approximates of how likely respondents analyze and respond to similar type of problem situations. The problem contains logical subtleties that may discriminate between those who have more developed KCS or those who can analyze problems correctly and those who still struggle to become proficient problem solvers. The problem represents a myriad of problem types that elicit responses that enable people to determine those who can analyze problem situations properly and those exhibiting alternative analyses.

Before solving the problems, the respondents were instructed to provide complete solutions and explanations for their answers so that the researcher can identify the nature of KCS manifested in their works. The outputs were collected, sorted, and classified according to the final answers given and corresponding solutions. Each group of answers and solutions were thoroughly analyzed to identify the nature of the knowledge constructions involved.

The different solution steps implemented were coded for easy reference and described in detail to ferret out the likely schemata used to guide the solution process.

Finally, the solution steps used by the respondents to answer the questions asked in the problem were evaluated to determine their level of problem solving engagements based on Polya's problem solving principles. Also, the researcher determined the respondents' nature of cognitive engagements based on Piaget's theory of cognitive development, particularly those of adaptation processes.

Results and Discussion

The problem situation involves both quantitative and qualitative sets of data. The set of quantitative data allows the framing of computational algorithms as well as equations that can be manipulated following some established procedures or steps in order to yield some desired information. The qualitative set of data provides information about the situation that may guide the solvers regarding the feasibility of a particular solution step or usability of a result.

The quantitative set of data includes a) gas

consumption rate of the vehicle, which is 3 liters per 2 kilometers of travel, b) distance travelled (150 km.) from farm to city house, c) the maximum amount of gas (150l) that can be filled into the vehicle's gas tank, d) total amount of gas reserve (450l) that may be used for the travel, and e) amount of gas (100l) that can be contained in a basin.

The qualitative set of data includes a) the fully loaded vehicle, b) two basins that can be brought along by hanging them at the rear end of the vehicle, c) the basins are for storage purposes only, and d) the drum where the 450 liters of gas is stored which may be brought along if empty.

Solutions of the Respondents to the Given Problem Situation

The solution steps provided by the solvers mainly involved algebraic procedures, such as forming equations. These are replicated below and are labeled A, B, C, D, and O.

Solution step A shows a procedure for determining the distance (x) that can be covered with 150 liters of gas.

(A)
$$\frac{150l}{x} = \frac{3l}{2km}$$
 $x = \frac{(2km)(150l)}{3l} = 100km$

Solution step B shows a procedure for determining the supposed farthest distance (x) that can be covered with 450 liters of gas.

(B)
$$\frac{450l}{x} = \frac{3l}{2km}$$
 $x = \frac{(2km)(450l)}{3l} = 300km$

Solution step C is a procedure for determining the amount of gas (x) that can be consumed in travelling 150 km.

(C)
$$\frac{150km}{x} = \frac{2km}{3l}$$
 $x = \frac{(3l)(150km)}{2km} = 225l$

Solution step D is a procedure for determining the amount of gas (x) needed to refill the gas tank after the original content of 150 liters is used up in order to reach the farmer's house in the city.

(D) 225l - 150l = 75l - amount of gas needed to reach the farmer's house



Solution step O represents all solutions steps that do not present logical procedure in obtaining a particular result, or justifying a particular conclusion. Some of these are replicated below.

 (O) : 150km/2km = 75km; 450l/3l = 150l. Therefore, the farthest distance that the man can travel is 150 km.

$$: 150l = \frac{2km}{3l} = \frac{450}{2} = km = 225km$$

Therefore the farthest distance that can be covered with 150 liters of gas is 225km.

- : Because 3*l* of gas is needed to cover 2km, then 150l is needed to cover 50km. Also, the farthest distance that the man can travel is 150l + 450l = 600l which is needed to cover 200km. Therefore the man can reach his house in the city.
- : The man can travel up to 50km, but he can reach his house in the city by walking or riding other vehicles that pass by.

Nature of the Solvers' Problem Solving Engagements in View of Polya's Problem Solving Principles

The main questions in the problem situation involve 1) finding the farthest distance that the farmer can travel, and 2) determining whether the farmer will reach his house in the city.

Figure 2 shows the level of problem solving engagements of the solvers in view of Polya's problem solving principles.

It shows that the level of problem solving engagements of the solvers is confined within the first stage, which is "understanding the problem." In this stage, solvers are expected to make sense of the problem situation and interpret or give meaning to the data based on the specific context described in the problem. The result of this stage should be used to device a solution plan up to the last stage, which is yielding the final answer. However, in the case of the respondents of the study, the different solution steps (A,B,C,D) they performed were only verification steps and constituted the initial steps needed to be done to figure out how to go about the more important procedures to solve the problem. For instance, solution step A only shows whether 150 liters of gas are sufficient to cover a distance of 150 km. Solution step B only shows how far 450 liters of gas could go. Solution step C only shows the amount of gas needed to cover 150 km., and solution step D only shows the amount of gas needed to be added to 150 liters in order to reach the farmer's house in the city. Clearly, the results derived from each and the combinations of all of these solution steps are only bits of information that could be used as input in devising a strategy to solve the problem.

It appears that all solutions only revolved around the set of quantitative data, which were processed using some algorithmic solution steps such as forming algebraic equations. The set of qualitative data were ignored as these were not considered in the solutions. This result suggests that the existing problem solving schemata of the solvers would entertain only quantitative data, perhaps because these data could be fitted to procedural solution steps that they are

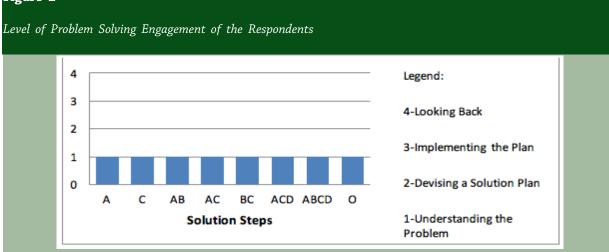


Figure 2

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likely accustomed to doing and which fit their framework of thinking.

Nature of the Solvers' Cognitive Engagement in View of Piaget's Adaptation Processes

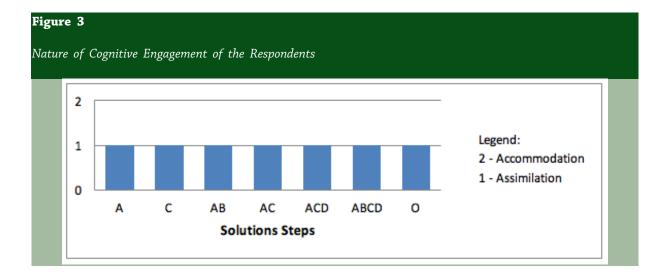
Figure 3 shows the nature of the solvers' cognitive engagements in view of Piaget's theory of cognitive development.

Based on the solution steps performed, it can be deduced that the solvers' cognitive engagement in problem solving involved mainly assimilation. They are simply using their existing procedural knowledge of solving a routine problem to the new problem situation, which is not routine. As a result, they were only able to do the initial steps of the whole solution process. It appears that their existing problem solving schemata would tell them that their procedures constituted the totality of the solution, and that is why they ended their solution steps where quantitative data ended to be algebraically manipulated. There were no coherent solution steps that showed any departure from algebraic procedures. A solution that attempted a procedure other than algebraic one had to dwell on the assumption that is impossible in the problem situation. These results support the problem raised with schemata (Cherry, 2016) in which people tend to cling to their existing mental framework of doing things, in this case, problem solving. They continue to do what they have been used to do, instead of adapting to the new problem situation.

The exclusion of qualitative data in the solvers' solution steps appears to be an obvious indication that said data do not fit to their existing problem solving schemata, and this is why these are ignored or not given importance and meaning in their solution steps. As a result, they treated the problem situation like any ordinary routine problem which can be solved using certain algorithmic procedure. Also, the results somehow support similar schema issue as pointed out by Nadkarni and Narayanan (2007) that people tend to preserve their existing framework of thoughts or schemata by "re-interpreting new information or distorting them to fit." In this case, respondents became selective on the type of data (only quantitative data) that they interpreted.

If the solvers considered the set of qualitative data, they could have realized that the solution steps they used are only preliminaries that serve to guide their supposed exploratory attempts, particularly in framing implementable solution strategies. They could have realized that some of their solution steps are not feasible, and hence could not be used to answer the question asked in the problem. Also, they could have realized that some of their assumptions that guided their solution steps should not have been considered at all.

If the solvers have considered both quantitative and qualitative data, they could have fully appreciated the depth of the problem situation and could have led to problem solving engagement that requires exploratory attempts. They could have seen how their existing knowledge of facts and procedures, and skills



in mathematical modeling a problem situation can be used more productively. In this case, they could have properly interpreted the results of their solution steps in the context of the problem situation. For instance, the result in step A solution informed the solver that 150 liters of gas can only allow 100 km distance of travel and therefore was not enough to reach the house. The result in step B solution informed the solver that 450 liters of gas could allow 300 km of travel but on condition that the given amount of gas can be brought along. The result yielded in step C solution informed the solver that 225 liters of gas were needed to reach the house in the city. The result in step D solution informed the solver that there is a need to refill the vehicle's gas tank with 75 liters more in order to reach the city house but on the condition that refilling of gas be done along the way.

Also, the solvers could have observed the meaning and importance of the qualitative data. For instance, the information that "the vehicle is fully loaded" should indicate that it is impossible to bring along any extra amount of gas aside from that in the gas tank. The information that "there are two basins which can be brought along" indicates that in case the farmer decides to store gas along the way, then this option is possible. The information that "the drum which contains 450 liters of gas can be brought along when empty" indicates that in case the farmer decides to use all his gas reserve and needs to bring these along the way, then this option is also possible.

Thus, when both kinds of data are considered together, it becomes apparent to the solver that steps B and D are not feasible solutions because extra amount of gas cannot be brought along. Also, the solver could have been led to consider a solution strategy of using up all 450 liters of gas by travelling short distances back and forth until such time that the remaining amount of gas is enough to reach the house in the city. For instance, having all 450 liters of gas requires filling the gas tank three times and entailing five one-way trips. To make this happen, the farmer must deposit some amount of gas at a certain distance along the way that would make possible the accumulation of exactly two full tanks of gas. In turn, this remaining amount must be brought to a certain distance that would permit the accumulation of exactly one full tank of gas. It is only at this moment when the solver would know whether the remaining one full tank of gas is sufficient to cover the remaining distance to reach the house in the city. The problem of knowing the exact distance at which exactly one or two full tanks of gas are accumulated can now be solved using algebraic procedure customized for this particular situation.

At this juncture, it may be said that the very reason why solvers could not advance in their solution steps beyond the verification stage is that they have not considered or have not found meaning in the qualitative data, which could have opened a new gate for understanding or a new avenue for exploratory attempts. It is for the same reason that the solvers' problem solving efforts were stifled and fared poorly against the reflective aspect of Polya's problem solving principles.

Implications of Results to Mathematics Education

The results indicate that the respondents somehow have the propensities for looking at a real-world non-routine problem situation as mathematically solvable, which is a commendable thought process expected of a mathematics teacher. It has been mentioned in literature that an important aspect of good mathematical thinking is having the predilection of looking at any problem situation as potentially solvable using mathematical procedures. However, a caveat to remember is not to fall to the tendency of looking at the solution of the problem as consisting mainly of procedural or algorithmic steps. The respondents of the study very well demonstrated this tendency. This is why their problem solving engagements were superficial, making opportunities for exploratory attempts unchartered. The instances where remain solvers just proceeded with their solution steps without ever using the results to answer the questions in the problem are clear indications of poor practice of the reflective or metacognitive component of problem solving. Also, the instances where solvers either put up solutions steps based on impractical assumptions or simply implement illogical solution steps indicate poor exposure to real world non-routine problem solving. This result implies that the Philippine Mathematics education is not reinforcing the development of abilities to solve non-routine problem situations.

Since the respondents are all mathematics

teachers. the solutions they demonstrated somehow reflect the kind of KCS that the Philippine educational system has developed in them during their school years. These schemata, characterized as rigid and procedural framework of thought, appear to be the dominant lenses the respondents use to interpret and solve a problem situation. It is more likely that such kinds of thought will be the ones to be transferred to their students. This means that until the country's educational system ventures towards a paradigm shift in its approach to Mathematics education, it is unlikely that students will develop a kind of mathematical thinking, or KCS, that are flexible, reflective, and transformational. Transformational thinking is the "ability and disposition to engage in authentic real-world problem solving" (Jurdak, 2016). Accordingly, "it requires metacognitive processes that lead to profound changes in problem solving practices." It allows solvers to see world problem situations as potentially like school mathematics problem situations. Hence, they can apply more fruitfully their mathematics knowledge and skills in such a new context.

Based from the results, for as long as mathematics education continues to pursue a framework designed mainly to tackle routine problem situations, students are likely to develop mathematical KCS similar to that demonstrated by the respondents of this study, which are mainly procedural, superficial, and one-dimensional. Thus, it is in order to rethink the Philippine school mathematics education to include the ability to solve real-world non-routine problem situations as an essential learning goal. After all, what is the importance of knowledge and skills developed in the classroom if these could not be used to facilitate the resolution of problems that learners encounter in their real-life situations?

Conclusions

The KCS of the respondents are characterized as rigid procedural frameworks of thought. These kinds of schemata appear to limit the way solvers interpret problem situations to mainly within the context of quantitative data. In particular, the problem solving engagements of the respondents revolved within the first stage of problem solving in view of Polya's problem solving heuristics. This level of engagement is only superficial and one-dimensional. It makes exploratory attempts unchartered.

Also, the nature of the solvers' cognitive engagements in view of Piaget's theory of cognitive development, particularly adaptation processes, is confined to mainly assimilation process, which, in this case, is possibly responsible in keeping the solvers from making exploratory attempts that could have paved the way towards attaining more productive problem solving. The influence of routine type of problem solving is so entrenched that solvers could not find meaning in qualitative data and has led to some alternative solution steps that do not necessarily address a problem situation. The ability to make necessary adjustments (accommodation process) in response to new problem situations remains a great challenge among the teacher - solvers.

Recommendations

Problem solving may be made the driving force and context of learning math in the classroom so that learners could clearly see the connection between math concepts and their uses in solving real-life problem situations.

There is a need to rethink how real-world non-routine problem solving may be integrated into school mathematics in order to develop among the students flexible, reflective, and transformational knowledge construction schemata. The development of such kind of framework of thought is an important aspect of cognitive development that education brings to learners for them to be able to deal with real-life situations more effectively.

Training programs on problem solving that deal with real-world problem situations may be developed for teachers to handle such type of problems effectively.

If possible, teachers may adopt а problem-based approach of teaching any mathematics subject to develop among the students not only problem solving skills but also and more importantly, appropriate problem solving schemata, which will enable them to interpret problem situation correctly.

References

- Cherry, K. (2016). What is a Schema in Psychology? Verywellmind. https://www.verywell.com/whatis-a-schema-2795873
- Cherry, K. (2017). What is a Schema in Psychology? Verywellmind. https://www.verywell.com/whatis-a-schema-2795873
- DiMaggio, P. (1997). Culture and Cognition. Annual Review of Sociology, 23: 263-287. https://doi.org/10.1146/annurev.soc.23.1.263
- Hein, G.E. (1991). Constructivist learning theory. *Exploratorium*. https://www.exploratorium.edu/ education/ifi/constructivist-learning
- Jurdak, M. (2016). Learning and teaching real world problem solving in school mathematics: A multiple-perspective framework for crossing boundary. Springer international publishing Switzerland.
- McLeod, S. (2018). Jean Piaget's theory of cognitive development. *SimplyPsychology*. https://www. simplypsychology.org/piaget.html
- Nadkarni, S., & Narayanan, V.K. (2007). Strategic schemas, strategic flexibility, and firm performance: The moderating role of industry clock speed. *Strategic Management Journal*, 28 (3): 243-270. https://doi.org/10.1002/smj.576
- Orleans, A.V. (2007). The condition of secondary school physics education in the Philippines: Recent developments and remaining challenges for substantive improvements. *Australian Educational Researcher*, 34(1):33-54. https://link. springer.com/article/10.1007/BF03216849
- Piaget, J., & Inhelder, B. (1958). The Growth of Logical Thinking From Childhood To Adolescence. London: Routledge, https://doi. org/10.4324/9781315009674
- Polya, G. (1957). How to Solve It. Princeton University Press, USA. https://math.hawaii.edu/ home/pdf/putnam/PolyaHowToSolveIt.pdf
- Schoenfeld, A. (1994). Reflections on doing and teaching mathematics. In A. Schoenfeld (Ed.).

Mathematical Thinking and Problem Solving. (pp. 53-69). Hillsdale, NJ: Lawrence Erlbaum Associates

- TIMMS & Pirls International Study Center. (2003). Average Achievement in the Mathematics Content Areas. *TIMSS & PIRLS International Study Center*, Lynch School of Education, Boston College. https://timss.bc.edu/PDF/t03_down load/T03_M_Chap3.pdf.
- Wadsworth, B.J. (2004). Piaget's theory of cognitive and affective development: Foundations of constructivism. Longman Publishing.